**Ability Grouping and Mathematics Education**

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***Introduction***

Ability grouping in education has long been the subject of debate. The discussion is particularly relevant to middle and secondary mathematics education, as mathematics, more than any other subject, tends to be taught in homogeneously grouped classes (Loveless, 1998; Boaler, Dylan & Brown, 2000). Many studies of ability grouping have focused on questions of equity, and the negative effects on children who are taught in low-ability classes (Slavin 1990; Oakes, 1995). Other studies have found that some grouping systems have benefits for learners (Kulik & Kulik, 1992). This paper focuses on the effect on students of ability grouping in mathematics classes, and what the implications of such groupings are for education in New Zealand.

In the commonly used term 'ability grouping', 'ability' refers to ability as perceived by teachers, or to achievement on a standardised test. In this paper the term 'ability grouping' refers to any of a variety of forms of homogeneous classroom ability groupings. In streaming, students are grouped for all classes. In setting, they are grouped by subject. Tracking may describe either system. Some schools group only for mathematics classes (Boaler et al., 2000). In Britain, the trend has been towards increased setting (Boaler et al., 2000). In the U.S.A, grouping by subject has become more typical than streaming (Loveless, 1998). In France, ability grouping is not permitted (Greenaway, 1999), while the British government has endorsed and promoted ability grouping for the past decade (Andrews, 2001).

The alternative to homogeneous classes is mixed-ability, or heterogeneous classes. In New Zealand, streaming is the most common form of ability grouping, and New Zealand secondary schools cover the range from tightly streamed (each of 15 different classes at the same year level being a different stream) to completely heterogeneous mixed-ability classes. Many schools use broad-band streaming, which is streaming for core subjects into parallel classes at three or four ability levels.

**Studies which support ability grouping in mathematics**

A meta-analysis by Kulik & Kulik (1992) of the effects on achievement of grouping programmes showed that programmes which offer the same basic curriculum have little or no effect on achievement, but that programmes differentiated for the aptitude of the group are beneficial for pupils of all ability levels. They found that, in ability-grouped classes, overall self-concept diminished slightly for above average students, but improved for low-ability students. Ireson, Hallam & Plewis (2001) studied 45 English secondary schools, and found that overall self-concept was higher in schools with moderate levels of ability grouping than in tightly streamed or un-streamed schools. They found that mathematical self-concept was not related to ability grouping in mathematics. Even some opponents of streaming agree that gifted children (the most able 3 to 5% of students) benefit from grouping in programmes designed to accelerate their learning (Rogers, 1991; Braddock & Slavin, 1995).

Within an ability-grouped class, students are able to contribute more equally to group work (Mills & Durden, 1992), and discuss ideas together more easily (Koshy, 2001). Meanwhile teachers struggle with the complexity of heterogeneous classes, and find it easier to teach ability-grouped classes. Since teachers often opt to teach to the 'average' child in the class, ability grouping achieves a better match between learning tasks and student aptitudes (Koshy, 2001). In America, Epstein & MacIver (1992, cited in Loveless, 1998), found that 8th grade algebra students of all ability levels did better in ability grouped classes, but that 8th grade mathematics survey students did better in mixed classes. Fuligni and Eccles (1995), in a study of 1,139 Michigan 7th grade students, found that high and medium ability students showed long-term benefits from being placed in an ability grouped mathematics class, but that there was no significant difference for low ability students. There is evidence that teachers tend to feel more positive about teaching same-ability mathematics classes, even if they have experience in mixed-ability teaching in a supportive environment (Dar, 1985, in Linchevski & Kutscher, 1998).

**Studies which support mixed ability mathematics classes**

 A meta-analysis by Slavin (1990) found that there were no significant positive effects of ability grouping for any programmes, except acceleration for the gifted. He excluded from his study those programmes which offered different curricula for different ability levels. Braddock & Slavin (1995) cite several studies that show negative effects of ability grouping, especially on students in low-ability groups who are frequently taught by less able teachers, cover less content than higher ability classes, and suffer from loss of motivation and self-image. In a study of three English secondary schools, Boaler et al. (2000) found many negative effects on ability grouping for the presumably able students in the highest set mathematics class. These effects included being taught at a pace too fast for students to develop understanding of what they were learning, and being taught too prescriptively. Teachers expected ability-grouped class members to all work at the same pace. The students in the top class were more negative about mathematics and were more likely to believe that memorisation was more important than thinking in mathematics, compared to lower or mixed-ability classes. Boaler & William (2001), in a study of 1000 students in six London schools, report a number of observed and reported disadvantages for students in lower set classes, including lower expectations, limited curriculum, an emphasis on rote learning and copying, and being taught by non-specialist teachers. Mixed-ability classes were more likely to be given differentiated tasks, or tasks with variable outcomes. Students in heterogeneous classes were more likely to be allowed to work at their own pace.

A study comparing mixed-ability to same-ability 7th and 8th grade mathematics classes in one Israeli school found that there were significant losses for middle and low ability students in the same-ability classes, and insignificant gains for high-ability students (Linchevski & Kutscher, 1998). In their study of 45 English schools, Ireson, Hallam, Hack, Clark & Plewis (2002) found that there were gains for high-achieving students in streamed classes, and slight losses for low-achieving students. International studies have pointed out the effectiveness of Japanese mathematics teaching where classes through elementary and middle school are mixed-ability and socially diverse (Okano & Tsuchiya, 1999, Schaub & Baker, 1994, Stigler & Hiebert, 1997).

**Resolving the ambiguity**

 One of the problems in synthesising the seemingly contradictory evidence on ability grouping is that the situation is very complex. Apart from the grouping system, classes differ in size, ability range within the class, teaching approaches, teacher competence and attitudes, curriculum, and resources. There can be a considerable range of ability, even within a 'gifted' mathematics class (Kalchman & Case, 1999). The lack of definitive evidence favouring either same-ability or mixed-ability grouping has led to calls for large scale, long-term studies (Mosteller, Light, & Sachs, 1996). The passion brought to the argument by researchers on both sides tends to impede clarity of discussion. The case for mixed ability is often put on the grounds of equity alone, rather than arguing its effectiveness for students of different abilities (Oakes, 1995). An obvious bias lends doubt as to the objectivity of some research. For example, the Linchevski & Kutscher (1998) study, cited above, states that "one of the project developer’s main goals" was to demonstrate that placement of students in mixed-ability classes was not detrimental (p 550).

There are some common threads evident in many of the studies. Most studies agree that high-ability students benefit from working with other high-ability students, where the teaching approach is tailored to their needs. Those who favour ability grouping point out that the only reason for ability grouping is to allow different teaching approaches, so that studies of grouping that use the same teaching approach for differently grouped classes are meaningless (Kulik & Kulik, 1992). However, many studies point to a negative effect of streaming on low-ability students, linked to the low expectations of the teacher, the teaching approaches used with these classes, and the social effects of being labelled as low achievers. These findings tie in with the New Zealand observations of Jones (1991), Kealey (1984), and Wood (1992), who describe how the behaviour of many students in lower streams can make it difficult for a teacher to teach effectively.

Both sides of the debate tend to compare the weaknesses of the opposing system to their own best practise. Evidence suggests that poor outcomes for students in low streamed classes are linked to administrative, attitudinal, behavioural and social factors, which means that low-stream classes are possibly more likely to encounter poor teaching than high streams. The theoretical advantages of suiting teaching to the level of the students, and making the job of the teacher easier, may, in fact make the teacher less apt to meet the needs of the range of ability levels in the class. Overall, though, there is considerable evidence accumulating for each side of the debate. The complexity of the situation means that research must go beyond ability grouping, and look at the factors that make ability grouping effective or ineffective within a classroom.

**Implications for teaching**

Teachers usually have no choice about teaching streamed or un-streamed classes. They fit into the system of the school at which they teach. Given the unresolved nature of the debate, it would seem premature to put energy into attempting to change the ability grouping system in New Zealand schools. What teachers can do is to recognise that every class has a range of ability within it, and teach in a way that caters to a range of ability. Some instructional approaches, which are put forward as alternatives to ability grouping, are actually independent of grouping, and can be implemented in either system. Viewing instructional practices as inevitably linked to either same or mixed ability classes may prevent schools from implementing improvements to their current system. Given that decades of debate have failed to show major effects of either same-ability or mixed-ability grouping, it is likely that there are more important factors determining how mathematics is learnt. It may be more productive to research ways to improve the teaching of mathematics in any classroom.

The research on mathematics teaching has found a number of effective instructional approaches. These may be divided into three categories: within-class grouping; differentiation; and fostering mathematically creative thinking.

**Within-class grouping**

One method of grouping students is by placing students into small teams within classes, based on either ability, or a random distribution. A meta-analysis of within class grouping found that students learning in groups of three or four within a classroom were found to achieve significantly more than students not learning in small groups, especially in mathematics and science classes, but this was only found to be true in traditionally taught classes or for individualised mastery learning (Lou et al., 1996). For experiential learning, there was no benefit of within-class grouping. In mathematics classes, there was no overall difference between homogeneous and heterogeneous groups. High ability students did equally well in each group, but medium ability students did better in homogeneous groups, and low ability students did better in heterogeneous groups. In heterogeneous groups, high ability students may do most of the work, so that average students have less opportunity to think and learn (Winebrenner & Devlin, 2001). Lou et. al. (1996) cited several studies that had found positive benefits for homogeneous within-class groupings, due to improved communication, and greater group cohesiveness. A study of computer-based mathematics instruction found that high ability students did better in homogeneous pairs than in heterogeneous pairs, or alone (Hooper, 1992).

Since there is considerable diversity even within same-ability classes (Boaler et al., 2000; Kalchman & Case, 1999), there is potential for both heterogeneous and homogenous groupings to be used, even within ability-grouped classes.

Co-operative learning is a teaching approach that uses within-class groupings in which students work together to learn, and are jointly responsible for their learning. Slavin (1995) argues that, in co-operative learning, team rewards and individual accountability are important for ensuring achievement. Co-operative learning may be used in either heterogeneously or homogeneously grouped classrooms (Mills & Durden, 1992).

**Differentiation**

Boaler et al. (2000) and Boaler & William (2001) found that many of the problems experienced by students in ability-grouped classes stemmed from the failure of teachers to differentiate among students, expecting them to learn at the same pace. Some forms of differentiation which have been advocated for high ability mathematics students are cluster grouping, acceleration, curriculum compacting, and enrichment. Advocates of mixed-ability schooling emphasise that the high expectations and enrichment opportunities that characterise programmes for the gifted will benefit all students (Renzulli, 1994; Wheelock, 1992). They recommend a highly differentiated teaching approach for all students, and emphasise the need for teachers in such a system to have more time for professional development and collaborative planning in preparation for lessons.

Considered broadly, differentiation is the provision of different curricula, resources, assessments and/or teaching approaches to different students. Mathematically able students in a mixed ability class require instructional and curricular modifications which meet their learning needs (Johnson, 2000). One method of group differentiation is cluster grouping, which brings together a group of three to six same ability students within a mixed-ability classroom (Winebrenner & Devlin, 2001). It offers students the opportunity to work together, while enabling one teacher to meet the learning needs of the whole cluster. Differentiation can also occur through the teacher providing self-instructional material or special projects, by individualised education programmes, or by pullout programmes (Holton & Gaffney, 1994).

Acceleration, or the completion of a standard curriculum in a shorter than usual time, is one form of differentiation. Many studies have shown that programmes which accelerate mathematically able students result in rapid and proficient learning (Sowell, 1993). Curriculum compacting is an individually differentiated approach in which pre-tests determine which parts of the curriculum have already been mastered, providing able children with class time for other learning.

Enrichment programmes provide extra opportunities for breadth and depth of learning alongside the regular curriculum. Rogers (1991) claims these programmes result in substantial academic gains in general achievement, critical thinking and creativity, while Sowell (1993) found that results were inconsistent, with some studies showing gains in achievement.

**Mathematically creative thinking**

*Mathematics in the New Zealand Curriculum* (Ministry of Education, 1992) emphasises the importance of creativity in mathematics. Research into the Japanese education system, which results in substantially higher levels of mathematical achievement than that shown by western countries, indicates that a key point of difference is the Japanese emphasis on mathematical creativity in primary and middle schools (Stigler & Hiebert, 1997). Most Japanese middle school mathematics teachers emphasised mathematical thinking as the main goal of their lessons, while United States and German teachers emphasised the learning of skills. The Japanese students spent most of their time inventing, analysing, and thinking, while United States and German students spent almost all of their time practising skills. Japanese classes are almost entirely of mixed ability. The implication could be that teaching mathematics with an emphasis on creative thinking will improve outcomes for students of all levels of ability, no matter how they are grouped in their classes.

**Implications for New Zealand**

Within the last decade there has been no published New Zealand research on ability grouping. While it continues to be a hotly debated topic in the United States and Britain, it is not an issue that promotes discussion here, yet ability grouping is increasing in Auckland secondary schools. The most recent EducationReview Office reports of Auckland state secondary schools indicated the grouping practices schools used, and only 3 of 18 had mixed ability classes in year 9 and 10 (Education Review Office, 2003). The remainder had broad-band or tightly streamed classes. Three schools had recently introduced ability grouped classes. From year 11 onwards, most schools have de-facto ability grouping in mathematics, as lower ability students study 'maths applied' instead of mathematics. Hipkins & Vaughan (2002), in their study of the impact of the National Certificate of Educational Achievement (NCEA) on six New Zealand schools, report a concern that having different senior classes attempting a range of achievement standards will mean a return to streaming. It may be time for ability grouping to once again become a focus of research in New Zealand, as it was twenty years ago.

New Zealand 13-14 year olds performed relatively poorly in the Third International Mathematics and Science Study (United States Department of Education, 1999), while Japanese students were among the most mathematically adept. The research into Japanese mathematics teaching has found that, as well as focusing on creativity, Japanese teachers lecture to the whole class more than United States teachers do. United States students spent more time working alone or in small groups. Time spent on explaining new material and lecturing to the whole class was correlated with increased student achievement, while having students spend time working alone or in small groups was correlated with lower achievement. Effective United States classrooms were found to be similar to Japanese classrooms (Schaub & Baker, 1994). Japanese teachers spend considerably more time preparing lessons and working co-operatively to improve their teaching than United States teachers do (Stigler & Hiebert, 1997). Stigler and Hiebert conclude that the absence of a systemic approach to teacher development is the biggest long-term problem facing American education, a conclusion which could equally be applied to New Zealand education.

*Mathematics in the New Zealand curriculum* (Ministry of Education, 1992) recommends a problem-solving approach to develop flexible and creative mathematical thinking, yet that is often not reflected in the way mathematics is taught in New Zealand secondary schools. My observations, and anecdotal evidence from other mathematics teacher trainees visiting a range of Auckland secondary schools, suggest that traditional, algorithmic teaching is the norm in most mathematics departments. Even problem-solving is taught with an algorithmic approach. New Zealand mathematics teachers could learn from their Japanese counterparts.

**Conclusion**

There is an interesting divergence between the factors found to be beneficial to learning mathematics in Western countries, and the mathematical excellence achieved in Japan by other methods. The results from studies of mathematics teaching in Japan suggest that a focus on group size or composition may be a red herring if we are truly interested in improving the standard of mathematics achievement in New Zealand. With conflicting evidence for and against ability-grouping, further research could address the factors that result in successful learning, rather than simply comparing one system with another. A common thread from many studies cited in this review was the negative effects on students of being taught as if they were a homogeneous group (e.g. Boaler, William & Brown, 2000). Many studies, particularly that of Stigler & Hiebert (1997), also focused on the need for teachers to have the time to collaboratively plan effective lessons. Teachers in the classroom, however, need to focus on teaching the range of ability that will certainly be present. To do this teachers need to work towards improving the quality of their own mathematics teaching. Working collegially to set up a system of ongoing collaborative teaching improvement may be a good way to begin.

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